Teoria Ergódica Diferenciável, assignment 8: Hyperbolic geometry

Rules: This is a home assignment for October 27. Please bring me the solutions no later than November 3.

Exercise 8.1. Let γ be an isometry of a hyperbolic plane which has a bounded orbit. Prove that γ has a fixed point.

Exercise 8.2. Let $\mathfrak{D}$ be a triangle in a hyperbolic plane. Prove that there exist points A, B, C on three different sides of $\mathfrak{D}$ such that diameter of the set $\{A, B, C\}$ is bounded by a constant which is independent from the choice of $\mathfrak{D}$.

Exercise 8.3. An absolute triangle in Poincare plane is a triangle with all three vertices in the absolute. Prove that all absolute triangles are congruent.

Exercise 8.4. Let γ be an isometry of a hyperbolic plane. Prove that γ has a fixed point in the plane or in the absolute.